Primary Student Teachers' Diagnosed Mathematical Competence in Semester One of their Studies

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Data collected from a diagnostics mathematics test taken by some primary student teachers are reported. Student responses were analysed using the Dichotomous Rasch Measurement Model. Error analyses enabled the identification of main misconceptions. Findings showed students performed relatively well with basic computations and visually presented data but struggled with word problems. The more complex and abstract the language used, the more difficult it became, implying that the critical skills of interpreting mathematical concepts, representations, and language and problem solving require explicit remediation. Implications for primary teacher education are provided.

Professional Teaching Standards (NCTM, 2005; AAMT, 2006) prescribe requirements such as a deep understanding not only of the teaching and learning processes but also the specific discipline content. Shulman's (1986) teacher knowledge taxonomy included *subject-matter content knowledge*, *pedagogical content knowledge* and *curriculum knowledge*. Although curriculum knowledge is knowledge of curriculum programs and instructional materials (Chick, 2002), Shulman (1986) defines subject-matter knowledge as knowledge of both the substantive structure and syntactic structure. Transforming subject-matter knowledge and curriculum knowledge into pedagogical content knowledge conceptualises "the link between knowing something for oneself and being able to enable others to know it" (Huckstep, Rowland, & Thwaites, 2003). Ma's (1999) study illustrated the need for primary teachers to have *profound understanding of fundamental mathematics* in order to promote and extend student learning. Ball and Bass (2000) argued teachers should be mathematically competent in order to effectively address the diversity of student needs. Research (Shulman, 1986; Ma, 1999; Ball, & Bass, 2000; Huckstep et al., 2003) also showed teachers' content knowledge of the curriculum generally influences their selection of activities and mediation of meaning in the classroom. This paper focuses on the identification of the mathematical competence of a cohort of foundation and primary student teachers in their first semester. Mathematical competence is defined as the ability to solve a set of items, in a written test, based on the Samoan Ministry of Education, Sports and Culture's (MESC) *Primary and Early Secondary Mathematics (PESM) Curricula* (MESC, 2003). Each item is designed to contribute meaningfully to a measure of mathematical competence. Ideally, student teachers should be capable of solving these items by critically (a) interpreting mathematical concepts, multiple data representations, and language describing quantitative relationships, (b) transforming interpretations arithmetically and/or algebraically, and (c) synthesising relevant knowledge and procedures to generate plausible solutions. The presence of mathematical competence is assessed by the *quality of student responses* and *nature of errors*. Therefore, the focus questions for this paper are: (1) How reliable was the test as a tool to measure students' mathematical competence? (2) What are primary student teachers' main mathematical misconceptions?

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Methodology and Analysis

The mathematics diagnostic test (MDT1, Appendix A) consisted of thirty items, compiled (Mays, 2005) primarily from the TIMSS 1999-R mathematics paper (Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrostowski, & Smith, 2000) (code T in the first position) as these have reliability and validity data, and eight items from the misconception literature (code M in the first position). These include five mental computation items (code MMCT) on products of single digit numbers and decimals, percentage of two-digit integers, four-digit substraction, and adding unit fractions (McIntosh & Dole, 2000; Callingham & Watson, 2004) and items on ordering fractions (MFNS08), the student-professor problem (MALG14), and proportional reasoning (MGE029) (Thompson & Saldanha, 2003). The 38 items sampled the content areas of *MESC's PESM Curricula* – fractions and number sense (FNS), measurement (MSR), algebra (ALG), geometry (GEO) and data presentation, analysis (DPA) and probability (PRB) – and five cognitive domains: *knowing, using routine procedures, investigating and problem solving, and mathematical communication* (Mullis et al., 2000). To provide access to students' errors, all 38 items were left open-ended. MDT1 was used at an Australian regional university with different cohorts of primary student teachers (Mays, 2005). A total of 140 Samoan primary student teachers took MDT1. Responses were categorised *Correct*, *Incorrect* or *Blank* and analysed using the Dichotomous Rasch Measurement Model and QUEST software (Adams & Khoo, 1996). Error analysis counted error types by item and identified up to 3 most common errors. The Rasch Model examines only one theoretical construct at a time on a hierarchical "more than/less than" logit scale (unidimensionality). Rasch parameters, item difficulty and person ability, are estimated from the natural logarithm of the pass-versus-fail proportion (*calibration of difficulties and abilities*) whereas estimation of fit is measured by mean square (mean squared differences between observed and expected values) and t, infit and outfit values (*estimation of fit to the model*). Fit of the data to the model (infit t values (-2, 2)) and reliability of the test (around 1) are examined.

Results

Review and Reliability of the Mathematics Diagnostic Test

The Rasch Model theoretically sets the mean of item estimates at 0 before item and person estimates are calibrated. Infit t values showed all items (except TGEO17) fit the model. A 3.70 infit t value indicated erratic behaviour. An item analysis from QUEST showed a non-monotonic increase in mean abilities for the 3 response categories, suggesting TGEO17 (difficulty -0.26 logits) might be measuring something different. Item TMSR27 had a zero score, meaning it was too difficult and was not discriminating among students. Thus both TGE017 and TMSR27 should be revised in future testing. Using the (-2, 2) infit t-criteria on cases confirmed they all fit the model. Candidate 119 had a zero score, which meant the case was not contributing to the calibrations. Finally, to improve the data's fit to the model, items TGE017 and TMSR27 and Candidate 119 were excluded from the second analysis of 139 cases and 36 items (see Table 1). The person ability mean of -.95 logits suggested the test was hard. A standard deviation of 1.15 indicated the cases were more clumped around its mean whereas the items were more spread out. An item fit map showed all items fit the model hence establishing that the 36 items worked together consistently to define a unidimensional scale. The reliability indices for items (0.97) and cases (0.84) were both high (Bond & Fox, 2001) indicating the test produced a reliable measure of student teachers' mathematical competence of MESC's PESM curriculum.

Table 1
Second Analysis - Summary of Item and Person Estimates

Estimates	Mean	SD	SD (adj)	Reliability	Infit Mean Square		Outfit Mean Square		Infit *t*		Outfit *t*	
					Mean	SD	Mean	SD	Mean	SD	Mean	SD
Item	0.00	1.85	1.82	0.97	0.99	0.10	1.05	0.55	0.00	0.83	0.11	0.90
Case	-0.95	1.25	1.15	0.84	1.00	0.25	1.05	0.95	0.00	1.05	0.17	0.83

The item-person map (Figure 1) corroborates the high reliability indices with its hierarchical distribution of items (represented by item codes) on the right of the vertical line from the most difficult to the easiest, and distribution of cases (represented by 'X') on the left, with both distributions sharing a common logit scale (on the extreme left). The two distributions are not aligned, corroborating that the test was hard for this cohort; further evident from the presence of more difficult items than cases above 2 logits. The model predicts people have a 50% chance of successfully solving items with estimates within their ability band (ability ± standard error), better chances of succeeding with items below the band and less than average probability with items located above the ability band. Items in Figure 1 are spread horizontally along their QUEST-generated logit locations into 6 content areas to facilitate discussions.

Cognitive Developmental Hierarchy of Items

Figure 1 displays both an overall and content-specific cognitive developmental scale of mathematical competence. At the top-end are the most difficult items (>3 logits, TFNS31 and MALG14), which are complex, non-routine word problems on investigation, multi-step problem solving, and algebraically representing multiplicative relationships. At the lower end are the easiest items (<-3 logits) involving basic computation (TALG28 and MMCT01). Above average items but below the most difficult items, involve increasingly less complex, multi-step word problems on likely outcomes, rate, ratio and quantitative descriptions; application of students' fraction understanding and knowledge; interpretation of complex diagrams, and mathematical communication. Below average items but above the easiest items, involve routine procedures (computing/modelling equivalent fractions); mentally computing percentage; algebraically transforming descriptions; mental computation; pattern extension (numerical and geometric); solving simple geometric word problems and linear equation; substitution; and graph interpretation. In summary, there seems to be distinct stages of cognitive development from *basic computations* with whole numbers at the lower end, through to interpretation of *visual data representations* and *explicitly stated quantitative relationships* around the middle (0 logits), and increasingly *implicit and abstract quantitative relationships* towards the top-end. Success rates and some common errors are presented next from the most difficult items and then by content area.

Common Errors and Misconceptions

Most difficult items. Item TFNS31 (4.32 logits, 0.7% success) showed 51 different error types with 26% of the students multiplying the given quantities, 22% responding "*71, 6.5, 500, 3.25, 32.5, 0.1, 0.05, or 1.2*" and 15% "*500/6.5*" with a 28.8% baulking rate. Item MALG14 (3.23 logits, 1.4% success) showed 55 different error types with 14% of the students responding "*16, 16:1* or *1:16*", 9% wrote "*y=16, n=16 or 16n*", and 6% gave "*16S=1P, x+y=16, 16/P, 16S/P, 16/x=n or A=16/n*" with a high 41.1% baulking rate. These errors suggested conceptual and computational difficulties.

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Item Estimates (Thresholds) all on all (N = 139 L = 36 Probability Level= .50)
Logits            |  Fractions        Algebra    Probability    Geometry    Measurement  Data Presentation
                  |& Number Sense                                                           & Analysis
5.0               |
                  |
                  |
                  |
                  |TFNS31
4.0               |
                  |
                  |
                  |
                  |                  MALG14                                          Most Difficult Items
3.0               |
                  |
                  |
                  |                                 TPRB10
                X |TFNS09            TALG18
2.0               |TFNS24
                  |MFNS08
               XX |
                X |
                  |
               XX |                  TALG38                                                 TDPA16
1.0             X |                  TALG33       TPRB13                  TMSR15
             XXXX |                                                       TMSR34
                X |
            XXXXX |TFNS30 TFNS35
           XXXXXX |TFNS06 TFNS26                  TPRB25
 .0      XXXXXXXX |TFNS32                                                                Item Mean = 0.00
           XXXXXX |
        XXXXXXXXX |TFNS22 MMCT02
          XXXXXXX |                  TALG11
         XXXXXXXX |MMCT05
      XXXXXXXXXXX |                                                                 Ability Mean = -0.93
-1.0      XXXXXXX |
        XXXXXXXXX |MMCT04          TALG07 TALG37
        XXXXXXXXX |                                           TGEO20
         XXXXXXXX |                  TALG21
              XXX |                  TALG19                   MGEO29
-2.0        XXXXX |
              XXX |MMCT03                                     TGEO23                         TDPA12
             XXXX |                  TALG36
            XXXXX |
            XXXXX |
                X |                                                                        Easiest Items
-3.0        XXXXX |                  TALG28
                X |
                  |
                  |MMCT01
                X |
-4.0              |
                X |
                  |
                  |
                  |
-5.0              |
         Each X represents    1 student
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Figure 1. MDT 1 Item-Person Map.

Fractions and number sense. The hierarchical difficulty order showed the most difficult to be a multiplicative relationship word problem (average weight) followed by a cluster of items on speed and unit conversion, operation with fractions and ordering fractions. Above the item mean is a cluster of items on speed, ratio, ordering decimals, fraction area-model, and fraction of an amount. Below item mean are items on equivalent fractions and mental computations. The latter, in decreasing difficulty, included computing percentage, multiplying decimals, adding unit fractions, 4-digit subtraction and multiplying 1-digit integers.

For item TFNS09 (2.30 logits, 5% success), the three most common errors (from 64 different error types) were "*3x8 =24m/s*" from 24% of the students; 3% wrote "*3 km=8* *min*" and 1.5% responded "*3000/s*" with a 25% baulking rate. Item TFNS24 (2.11 logits, 5.7% success) showed 50 different error types. The three most common errors were "*38*" (15%), "$\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$," "$\frac{1}{3}+\frac{1}{4}=\frac{2}{7}$," "$\frac{1}{2}+\frac{1}{4}+\frac{1}{24}$" or "$\frac{1}{3}\text{x}\frac{1}{4}=\frac{1}{12}$" (14%), and "*8*" (6%) with a 21.3% baulking rate. Item MFNS08 (1.98 logits, 7.9% success), highest error rate of the test (87.9%), showed 16 different error types. The three most common errors were arranging numerators/denominators in ascending order as "$\frac{2}{3},\frac{3}{5},\frac{5}{6},\frac{7}{10}$" (50%) or descending order "$\frac{7}{10},\frac{5}{6},\frac{3}{5},\frac{2}{3}$" (19%) and "$\frac{7}{10},\frac{3}{5},\frac{5}{6},\frac{2}{3}$" (4%) with a 4.3% baulking rate. Errors suggested conceptual and computational difficulties.

Item TFNS30 (0.38 logits, 17.9% success) showed 38 different error types. The three most common errors were "*330-4.5*" (8%), "*330x4.5*" (8%), and "*330/4.5*" (3%) with a 35.7% baulking rate. Item TFNS35 (0.47 logits, 18.6% success) showed 40 different error types where the three most common errors were "*2:3:6*" or "*200:300:600*" (12%), "*2:3*" (4%) and "*200+300+600=1100*" (4%) with a high baulking rate of 41.1%. Item TFNS06 (0.36 logits, 25.7% success) had 39 different error types. The three most common errors were "*0.5, 0.25, 0.037, 0.125, 0.625*" in increasing (21%) or decreasing decimal places "*0.625, 0.125, 0.037, 0.25, 0.5*" (9%), and "*0.625, 0.5, 0.25, 0.125, 0.037*" (6%) with a 2.9% baulking rate. For item TFNS26 (0.26 logits, 25.7% success), the three most common errors (from 11 different error types) were "*3 squares*" (27%), "*8 squares*" (10%), and "*6 squares*" (6%) with a 14.3% baulking rate. For TFNS32 (0.13 logits, 27.1% success), the three most common errors (from 35 different error types) were "*$150*" (15%), "*$182*" (13%), and "*$234.20*" (3%) with a 11.4% baulking rate. Item TFNS22 (-0.19 logits, 33% success) showed 44 different error types. The three most common errors were an incorrect third equivalent fraction (10%), two incorrect fractions (7%) and "$\frac{3}{4},\frac{4}{5},\frac{5}{6}$" (3%) with a 12.1% baulking rate. Errors demonstrated fraction and place value misconceptions and computational difficulties.

Mental computations. Item MMCT02 (-0.20 logits, 35.7% success), one of two items everyone attempted, showed 28 different error types. The three most common errors were"*30% of 50 = 15%*" (40%), "$\frac{30}{50}\text{x}100=60\%$" (3%), and "*30*" (1.4%). Item MMCT05 (-0.66 logits, 44.3% success) showed 12 different error types. The three most common errors were "*0.3x0.3=0.9*" (41%), "*0.3x0.3=9*" (2.2%) and "*0.3x0.3=0.03*" (2.2%) with a 0.7% baulking rate. Item MMCT04 (53.6% success) showed 30 different error types. The three most common errors were "$\frac{1}{2}+\frac{1}{3}=\frac{2}{5}$" (9.4%) indicating fraction misconceptions, "$\frac{1}{2}+\frac{1}{3}=\frac{1}{5}$" (6%) suggesting mis-remembered procedures, and "$1\frac{1}{2}+1\frac{1}{3}=2\frac{5}{6}$" (4%) reflecting poor listening skills with a 0.7% baulking rate. Errors from item MMCT03 (-2.13 logits, 71.4% success) showed 23 different error types. The three most common errors were "*5113*" (4%), "*5003*" (3%), and "*4113*" (3%) with a baulking rate of 1.1%. Item MMCT01 (-0.63 logits, 89.3% success), one of two items everyone attempted, showed 8 different error types. The two most common errors were "*48*" (1.4%) and "*8x7*" (1.4%) which reflected poor knowledge of multiplication facts.

Algebra and problem solving. The algebra item hierarchy also reflected the cognitively more demanding non-routine word problems (multiplicative and additive relationships) at the top-end with simple word problems around the middle and routine procedures towards the lower-end. Item TALG18's (2.27 logits, 5.7% success) three most common errors (from 38 different error types) were "*24m*" (18%), "*15m*" or "*9m*" (16%) and "*12m*" (7%) with a 17.1% baulking rate. Item TALG38 (1.26 logits, 9.3% success) showed 39 different error types. The three most common errors were "$\frac{1275x51}{50}$" (7%), "*1275x51*" (6%) and "*275+51=1320*" (2%) with a high 41.4% baulking rate. Item TALG33 (1.09 logits, 14.3% success) showed 44 different error types. The three most common errors were "*57 females, 29 males*" (16%), "*86-14=72*" (14%), and "*86+14=100*" (9%) with a 15% baulking rate. Error responses from TALG11 (-0.54 logits, 38.6% success) showed 34 different error types. The three most common errors were "*12*" (12.2%), "*3*" (3%), and "*1/3x48 =16*" (2.2%) and a 12.9% baulking rate. Item TALG07 (-1.17 logits, 52.9% success) showed 50 different error types. The three most common errors were "*5*" (4%), "*5x7+6=41*" (2%), and "*n(7+6)=41*" (2%) with a 4.3% baulking rate. Item TALG37 (-1.17 logits, 49.3% success) showed 25 different error types. The three most common errors were "*21 blocks*" (4.3%), "*5 blocks*" (3%), and "*13 blocks*" (1.4%) with a 13.6% baulking rate. Item TALG21 (-1.56 logits, 57.9% success) showed 36 different types, and the three most common errors were "*x=2*" (3%), "*x=6*" (2%), and "$x=\frac{42}{18}$" (1.4%) with a 9.3% baulking rate. Item TALG19 (-1.68 logits, 62.1% success) showed 31 different error types with three most common errors being "$\frac{18}{15}$" (4.3%), "*15*" (2.2%), and "*3*" (2.2%) with a 6.4% baulking rate. Item TALG36 (-2.31 logits, 68.6% success) showed 18 different error types. Three most common errors were "*9, 12*" (6.4%), "*10, 12*" (2.2%), and "*10, 20*" (2.2%) with a 9.3% baulking rate. Item TALG28 (-3.17 logits, 83.6% success) showed 9 different error types. Three most common errors were "*3n*" (2.2%), "n^2" (1.4%), and "*1x1x1*" (1.4%) with a 3.6% baulking rate. Errors suggested conceptual and computational difficulties.

Probability. The item hierarchical order reflected the decreasing level of cognitive difficulty from likely outcomes and expected number to application. Item TPRB10 (2.54 logits, 3.6% success) showed 36 different error types. The three most common errors were "*head*" or "*tail*" (13%), "$\frac{5}{10}$" or "$\frac{1}{5}$" (12%) and "*5/2 or 2.5*" (10%) with a 31.4% baulking rate. Errors with TPRB13 (1.09 logits, 11.4% success) showed 35 different error types. Three most common errors were "$\frac{3000}{5}=600$" (10%), "*100-5=95*" (6%) and "*5x100=500*" (5%) with a 30% baulking rate. Item TPRB25 (0.22 logits, 23.6% success) showed 35 different error types. The three most common errors were "$1/11$" (11%), "$3/11$" (10%), and "$1/3$" (3%) with a 23.6% baulking rate. Errors implied conceptual and computational difficulties.

Geometry. The three items displayed a hierarchy of decreasing cognitive difficulty from calculating a missing angle and similar triangles to identification of a 45° angle. Item TGEO20 (-1.44 logits, 54.3% success), showed 24 different error types. The three most common errors were "*115+115+70=290; 360-290=70*" (6%), "*115-70=45*" (3%), and "*180-115=65*" (3%) with a 12.1% baulking rate. Item MGEO29 (-1.75 logits, 62.1% success) showed 17 different error types. The three most common errors were "*12–6*" (9%), "*10-6=4*" (7%), and "$\frac{1}{2}x5x6$" (6%) with a 7.1% baulking rate. Item TGEO23 (-2.20 logits, 72.1% success) showed 10 different error types. The three most common errors were "*R*" (>90°) (5%), "*Q*" (90°) (5%), and "*P*" (90°) (3%) with a 2.1% baulking rate, indicating forgotten basic geometric facts.

Measurement. The two items were basically the same difficulty level on interpreting data from nested geometric shapes. Item TMSR15 (0.93 logits, 17.1% success) showed 27 different types. The three most common errors were "*144*" (15%), "*64*" (14%), and "*96*" (11%) with a 6.4% baulking rate. Item TMSR34 (0.87 logits, 17.1%) showed 30 different types. The three most common errors were "*16*" (19%), "*12*" (16%), and "*15*" (2%) with a 13.6% baulking rate. Errors indicated conceptual and computational difficulties.

Data presentation and analysis. The hierarchical order of difficulty reflected the level of cognitive processing required to determine a pictograph scale and reading histogram data. Of the 35 error types counted for item TDPA16 (1.19 logits, 13.6% success), the three most common errors were "*51*" (18%), "*8*" (18%), and "*Orange: 6 and Lime:7 houses*" (11%) with a 9.3% baulking rate. Item TDPA12 (-2.06 logits, 70% success) showed 14 different error types with the three most common errors being "*5 pupils*" (17%), "*14 pupils*" (3%), and "*8 pupils*" (1.4%) with a 1.4% baulking rate. Errors indicated conceptual and computational difficulties.

In summary, for the fractions and number sense items, the highest error percentage for a single error type was ordering fractions using only the numerators/denominators (50%) followed by the product of 1-digit decimals as a 1-digit decimal (41%), the percentage of a number as a percentage (40%), and area-model of an equivalent fraction using only the numerator (27%). The next two highest error percentages were words problems where students simply multiplied given quantities for average weight (26%) and average speed (24%). For the probability items, the most common errors (≥10%) indicated misconceptions about likely outcomes, expected number and favourable outcomes. The most common misconceptions (≥5%) for the geometry items were about similarity and basic geometric facts. For the measurement items, the most common errors (15 and 19%) were conceptual difficulties extracting relevant information from diagrams while it was incorrect interpretation of the language of the problem and visual data (17-18%) with the data presentation and analysis items. Finally, if mastery of the mathematics content is set at 80% success rate, then mastery level was not achieved for the majority (94% or 34/36) of the items. Overall, two-thirds of the items were quite difficult as evident from the number of above-item-mean items and less-than-50% success rates. Also high baulking rates (41.1%) were noted for three items requiring critical interpretation of multiplicative descriptions and critical organization and synthesis of information (i.e., critical problem solving). Error analyses provided additional, empirical evidence of the nature and extent of students' content-specific misconceptions and computational difficulties.

Discussion

Rasch statistics established that the diagnostic test was a reliable test to produce a unidimensional, cognitive developmental scale for students' mathematical competence. The item-person map and success rates showed students found non-routine word problems with abstract, multiplicative descriptions the most difficult and basic computations the easiest. This general pattern was also reflected within each content-area. Error analyses provided further insights to the most common errors for each item. From the item-person map and error analyses, it appeared that, in addition to weak content knowledge, students generally demonstrated difficulties in three crucial ways, firstly, critically interpreting the meanings of mathematical concepts in word problems (*average weight*, *average speed*, *likely outcomes*, *ratio* and *perimeter*); mathematical representations (*pictographs*, *bar graphs* and *complex diagrams*); and mathematical language (*twice as long*, *14 more females than males*, *16 students to one professor,* and *more than 5 minutes*). Secondly, student teachers demonstrated difficulties critically transforming their interpretations *arithmetically* to obtain numerical values (geometric and numeric pattern extensions, relational description, and operations with fractions); and *algebraically* to communicate general rules (tabular pattern extension and student:professor). Thirdly, students demonstrated difficulties critically managing, selecting and organizing relevant information (i.e. problem solving skills) to generate plausible solutions. Computational errors were also evident after selecting an appropriate procedure (calculating the interior angle and operations with fractions). Finally, findings from this study of Samoan primary student teachers contribute to the literature on preservice teachers' mathematics content knowledge (Shulman, 1986; Ma, 1999; Mays, 2005; Ball, 2000, Chick, 2002) and further confirm findings reported by others on misconceptions with mental computations (Callingham & Watson, 2004) and word problems involving fractions, and multiplicative reasoning with ratios and proportions (Thompson & Saldanha, 2003).

Conclusions and Implications

Findings from this paper show student teachers' content knowledge of the primary and early secondary mathematics curriculum appears to be lacking in conceptual depth in some content areas. Students' main misconceptions may be a mutual interaction of weak: (1) content knowledge of the curriculum, (2) critical interpretation of mathematical concepts, multiple representations, and language of the problem, and (3) critical problem solving skills (CPSS). CPSS permeate and underpin (1) and (2). Student teachers exhibit poorly developed fraction and number sense such as in ordering fractions and decimals, modeling equivalent fractions, and operating with fractions and applying fractions in ratio and proportion (Thompson & Saldanha, 2003). Findings also suggest students have underdeveloped conceptual understanding of probability, weak knowledge of basic geometric properties (similar triangles, quadrilaterals, rectangles and angle types), and weak algebraic skills. Although they mastered mental multiplication of 1-digit whole numbers and simplifying basic algebraic expressions, solving word problems was difficult. As descriptions of quantitative relationships become increasingly abstract, implicit and multiplicative, students struggle to access the mathematics embodied in problem statements and visual representations whereas they cope better with simple word problems and basic computation. Student errors demonstrate poor critical problem solving skills to interpret and analyse given information effectively, represent, and synthesise relevant knowledge and appropriate procedures to generate correct responses. Since pedagogical content knowledge is dependent on subject-matter knowledge and curriculum knowledge, student teachers need to know the mathematics first as learners before they can teach others to know (Huckstep, Rowland, & Thwaites, 2003). Aspiring to become effective teachers of primary mathematics means being proficient problem solvers who are competent at mastery level with the content of the primary and early secondary mathematics curricula. This implies that explicit remediation of student teachers' identified misconceptions needs to form part of their teacher education courses to specifically enhance their content knowledge, and critical skills in interpreting mathematical concepts, multiple representations, and language used in problems.

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Appendix A

Text Descriptions of MDT1 items.

Item	Text Descriptions	Item	Text Descriptions
MMCT01	8 x 7 = ?	TGEO20	Missing angle of a quadrilateral given 70°, 115° and 115°.
MMCT02	What is 30% of 50?		
MMCT03	8006 – 2993 = ?	TALG21	Find the value of x if 12x – 10 = 6x + 32.
MMCT04	$\frac{1}{2}+\frac{1}{3}=?$		
MMCT05	0.3 x 0.3 = ?	TFNS22	Write three fractions equivalent to $\frac{2}{3}$.
TFNS06	Write in ascending order 0.625, 0.25, 0.037, 0.5, 0.125.	TGEO23	In the diagram, which angle has a measure closest to 45°?
TALG07	An unknown number *n* is multiplied by 7 and then 6 is added to the result. The final answer is 41. Write this as a mathematical expression.	TFNS24	Penny had a bag of marbles. She gave one third of them to Rebecca She then gave a quarter of the remaining marbles to Jack. If Penny ended up with 24 marbles, how many did she start with?
MFNS08	Write in ascending order $\frac{5}{6},\frac{2}{3},\frac{7}{10},\frac{3}{5}$.		
TFNS09	An athlete ran 3 kilometres in exactly 8 minutes. What was her average speed in metres/sec?	TPRB25	Eleven chips are labelled 2, 3, 5, 6, 8, 10, 11, 12, 14, 18 and 20 respectively. The eleven chips are placed in a bag

TPRB10	If a fair coin is tossed, the probability that it will land heads up is 1/2. A fair coin is tossed 4 times and it lands heads up each time. What is likely to happen when the coin is tossed a fifth time?
TALG11	If 4 times a number is 48, what is one third of the number?
TDPA12	The graph shows the time taken to travel to school by a group of students. How many pupils travel for more than 10 minutes to reach school?
TPRB13	A sample of 100 light bulbs is chosen at random from a complete batch containing 3000 voters. When the sample is tested, it was found to contain 5 faulty light bulbs. How many faulty bulbs would you expect to find in the complete batch?
MALG14	At a particular university, there is an average of 16 students to every professor. Write this as a mathematical equation.
TMSR15	A rectangular garden bed adjoins a building as shown in the diagram.The garden bed has a path on 3 sides. What is the area of the path?
TDPA16	Two streets in a town have 30 houses (Orange St.) and 21 houses (Lime St.) respectively. This is represented in the pictogram. How many houses are represented by the symbol?
TGEO17	Which two of the four triangles are similar?
TALG18	An elevator starts at the first floor of a building. It travels up to the fifth floor, then down to the third floor and back up to the fourth floor. If the floors are 3 metres apart, how far did the elevator travel?
TALG19	If x = 3, what is the value of $\frac{5x+3}{4x-3}$?
	and one is drawn out at random. What is the probability that the number on the chip is a multiple of 3?
TFNS26	Shade 3/8 in the given (6 x 4) grid.
TMSR27	The length of the rectangle is twice as long as it is wide. What is the ratio of the width to the perimeter?
TALG28	Write in simplest form $n \times n \times n$.
MGEO29	In the diagram (of similar triangles), what is the length of the interval BD?
TFNS30	Sound travels at approx. 330 m/sec. A lighting strike was followed 4.5 seconds later by a clap of thunder. How far away did the lightning strike?
TFNS31	A pile of salt contains 500 individual crystals and has a weight of 6.5kg. What is the average weight of a salt crystal?
TFNS32	Laura had $240 but spent five eighths of it. How much money did she have left?
TALG33	A club has 86 members with 14 more female members than male members. How many males and females are members of the club?
TMSR34	What is the area of shaded rectangle?
TFNS35	A fertilizer mix contains 200g of nitrate, 300g of phosphate and 600 g of potash. What is the ratio of the weight of the nitrate to the total weight of the fertilizer?
TALG36	Extension of a geometric pattern.
TALG37	Extension to 2 terms of a numeric tabular pattern based on ALG36.
TALG38	If we produced a figure with 50 rows, we would require 1275 blocks. Explain how to calculate the number of blocks required to construct a figure with 51 rows.

Number in the Item Code corresponds to the Item Number in MDT1.